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A MEANS OF MEASURING THE EFFECT OF WAVE FORMS UPON THE IRON LOSS IN TRANSFORMERS.

A MEANS OF MEASURING THE EFFECT OF WAVE FORMS UPON THE IRON LOSS IN TRANSFORMERS

A Thesis Submitted

By

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A MEANS OF MEASURING THE EFFECT OF WAVE FORMS UPON THE IRON LOSS IN TRANSFORMERS.

Introduction.

The iron losses in a static transformer consists of the loss due to hysteresis and the loss due to eddy currents. According to Steinmetz the iron loss is

$$W = \frac{K_h f V B^{1.6}}{10^7} + \frac{K_e (f t B)^2 V}{10^{11}}$$

where

W = total loss in watts;

K_h = coefficient of hysteresis;

K_e = coefficient of eddy currents;

f = frequency in cycles per second;

V = volume of iron in core, cu. cm;

t = thickness of lamination, cm;

B = maximum flux density, lines per sq. cm;

It has long been known that the maximum flux density, B, which is induced in the core of the transformer, is dependent upon the form of the wave of electrical pressure which is applied to the primary windings of the transformer. This may be demonstrated as follows:(1)

Let e = instantaneous value of primary voltage.

i = instantaneous value of primary current.

ϕ = mean value per turn of instantaneous flux linking the primary and secondary current.

n = number of turns on the primary

Φ = maximum value of flux.

(1) Russell's Alternating Currents Vol.II p.246.

t = time in seconds.

T = period of wave of e.m.f. in seconds.

A = area under one-half of the e.m.f. wave.

S = cross section of the core in sq. cm.

Neglecting the resistance and leakage drop, which in practice are quite small, the instantaneous value of e.m.f. may be written

$$e = n \frac{d\phi}{dt} 10^{-8}$$

From this formula it is seen that when e. is zero, $\frac{d\phi}{dt}$ is also zero, which means that the rate of increase or decrease of ϕ is zero, and therefore ϕ must be at either its maximum positive or maximum negative value. Because the maximum positive value of the alternating current which produces this flux is equal to its maximum negative value, the maximum positive and maximum negative values of ϕ are equal numerically but have opposite signs. Let e vanish when t is zero, $\frac{T}{2}$, T, etc., and we have

$$10^{-8} n \int_{\phi}^{\frac{T}{2}} \frac{d\phi \, dt}{dt} = \int_{0}^{\frac{T}{2}} e \;\; dt.$$

But e , integrated between the limits zero and T/2, is equal to the area under half the wave of the applied e.m.f.; and since ϕ is a maximum in one direction when t is zero and a maximum in the opposite direction when t is $\frac{T}{2}$, by substituting n $d\phi$ 10 for its equivalent e dt, we may write

$$n\,10^{-8}\int_{-\Phi}^{\Phi} d\phi = A$$

$$\int_{-\Phi}^{\Phi} d\phi = \frac{A\,10^{8}}{n} = \phi\Big[_{-\Phi}^{\Phi} = 2\,\Phi$$

$$\Phi = \frac{A\,10}{2n}$$

since $\Phi = S\,B$

$$B = \frac{A}{2nS} \times 10^{8}$$

This formula proves that the maximum flux density is directly proportional to the area of the wave of the applied potential difference. From the formula for the iron losses given by Steinmetz, the hysteresis loss is proportional to the 1.6th power of B and the eddy current loss is proportional to the 2nd power of B. Thus it is evident that the iron loss in a static transformer is dependent upon the area of half of the e.m.f. wave applied to the primary winding.

It is customary for manufacturers to sell transformers under a guarantee that the iron loss shall not exceed a certain value when the rated e.m.f. impressed across the primary is sine shaped. In ordinary commercial work the voltage wave is seldom sine shaped, and, since the area of a peaked wave is greater than the area of a sine wave when the effective value of the [illegible] waves are equal, [illegible] it is impossible to tell from the data obtained [illegible]

(1). [illegible]

$$n\ 10^{-8} \int_{-\Phi}^{\Phi} d\phi = A$$

$$\int_{-\Phi}^{\Phi} d\phi = \frac{A\ 10^8}{n} = \phi \Big[_{-\Phi}^{\Phi} = 2\ \Phi$$

$$\Phi = \frac{A\ 10}{2n}$$

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It is customary for manufacturers to sell transformers under a guarantee that the iron loss shall not exceed a certain value when the rated e.m.f. impressed across the primary is sine shaped. In ordinary commercial work the voltage wave is seldom sine shaped, and, since the area of a peaked wave is greater than the area of a sine wave when the effective value of the three waves are equal, (1) it is impossible to tell from the data obtained in the tests ordinarily

(1). Russell's Alternating Currents. Vol.I p.71

made on transformers whether, the iron loss is greater or less than guaranteed.

In this thesis an attempt has been made to devise a scheme whereby the hysteresis and eddy current losses for a sine shaped wave can be calculated from the data obtained in a test performed with an e.m.f. wave of any shape.

Theoretical

It has been proved that the maximum flux density, B, is proportional to the area under half the wave of impressed e.m.f., therefore,

$$\frac{A_o}{A} = \frac{B_o}{B} \qquad (1)$$

where the subscript zero applies to the case of a sine wave.

For a wave of any shape

$$A = x\,y$$

where x is measured from $t = o$ to $t = \frac{T}{2}$ and y is the mean ordinate measured to the same scale. The value of x depends on the number of cycles that the impressed e.m.f. passes thru per second, and for the same frequency it has the same value. The value of y may be changed by changing the value of the impressed voltage or by changing the form of the wave of impressed voltage. Since x is constant

$$\frac{y_o}{y} = \frac{B_o}{B} \qquad (2)$$

For a sine wave

$$y_o = .6366 \text{ e max.}$$

$$\text{and } E = .7071 \text{ e max.}$$

Where E is the effective value of the applied voltage as measured by a voltmeter.

$$e\ max. = \frac{y\ o}{.6366} = \frac{E}{.7071}$$

$$y_o = \frac{.6366}{.7071} = 0.9\ E$$

Therefore from (2), $\frac{0.9\ E}{y} = \frac{B_\Delta}{B}$ (3)

The method by which it was undertaken to measure the mean ordinate of the wave, was to rectify the alternating current by means of a cell with aluminum and carbon electrodes in a suitable electrolyte. As a cell of this type will allow a current to pass freely when the carbon is the positive terminal and will prevent the passage of a current when the aluminum is the positive terminal, it is evident, that the alternating current, which changes from a positive maximum to a negative maximum, will be converted into a direct current which changes from a positive maximum to zero, or, in other words, the negative half of the wave is cut off.

A direct-current voltmeter of the moving coil type, owing to the principle on which it operates, measures the mean potential difference between the points to which its terminals are connected, and if a cell of the type mentioned above is placed in series with the direct-current voltmeter across an alternating-current voltage, the cell will rectify the

voltage and the voltmeter will give the mean value of the rectified voltage. The voltmeter reading will not equal the mean value of the alternating-current voltage because there are losses in the cell and because there is a leakage of current when the aluminum electrode is positive. Therefore it is necessary to determine the relation between the voltmeter reading and the mean value of the alternating current voltage.

This may be done by taking wave forms of the alternating current by means of an on dograph and measuring the ordinates of one-half of one of the waves, and calculating the mean value and the M.R.S of the ordinates. Signifying the mean value of the ordinates by y_1 and the M.R.S by E_1

$$\frac{E}{E_1} = \frac{y}{y_1}$$

Let $K = \frac{E}{E_1}$

then $y = Ky_1$

If Ea is the value of the rectified voltage $K_1 = \frac{y}{Ea}$

Where K_1 is the coefficient that the direct-current voltmeter reading has to be multiplied by to obtain the mean value of the alternating current voltage

$$\text{Then } \frac{0.9\,E}{K_1\,Ea} = \frac{Bo}{B} \qquad (4)$$

If in a test, the hysteresis loss is separated from the eddy current loss, the losses for a sine shaped wave can be

calculated as follows:

$$\frac{W_{ho}}{W_h} = \left(\frac{B_o}{B}\right)^{1.6}$$

$$\frac{W_{eo}}{W_e} = \left(\frac{B_o}{B}\right)^{2}$$

and $$W_o = W_h\left(\frac{B_o}{B}\right)^{1.6} + \left(\frac{B_o}{B}\right)^{2} W_e$$

It is difficult to separate hysteresis and eddy current losses experimentally, but an approximate value of the losses for a sine wave may be calculated from the data obtained in a test made with an e.m.f. wave of any shape as follows:(1)

Let q equal the ratio of hysteresis loss to the total loss and assume that the eddy current loss is constant, then

$$\frac{W_o - W}{W} = q\,\frac{W_{oh} - W_h}{W} = q\left(\frac{W_{oh}}{W_h} - 1\right)$$

$$= q\left[\left(\frac{B_o}{B}\right)^{1.6} - 1\right]$$

$$W_o = q\,W\left[\frac{B_o}{B}\right]^{1.6} + W(1-q)$$

The ratio q may be determined approximately by runs at two frequencies using voltages proportional to the frequencies. One frequency f., and voltage should be those used in the test. Under these conditions the total flux is constant because the wave form of e.m.f. is not changed by

(1). Bulletin of Bureau of Standards, Vol. IV. p473.

by changing the speed of the alternator unless the field excitation is changed, and this is not necessary as the voltage varies with the speed. Hence B is constant and

(1) $W_1 = h\ f_1 B^{1.6} - a\ f_1^2 B^2$

(2) $W_2 = h\ f_2 B^{1.6} - a\ f_2^2 B^2$

Dividing by the frequency and substracting (2) from (1)

$$\frac{W_1}{f_1} - \frac{W_2}{f_2} = a\ B^2 (f_1 - f_2)$$

$$a\ B^2 = \frac{\frac{W_1}{f_1} - \frac{W_2}{f_2}}{f_1 - f_2}$$

$$W_1 - W_{h1} = f_1^2 a\ B$$

$$\frac{W_1 - q\ W_1}{f_1^2} = \frac{\frac{W_1}{f_1} - \frac{W_2}{f_2}}{f_1 - f_2}$$

Hence $q = \frac{\left(\frac{f_1 W_2}{f_2} - \frac{f_2 W_1}{f_1}\right) f_1}{(f_1 - f) W_1}$

Experimental

The first problem was to determine how fully the current could be rectified and the best electrolyte for the purpose. The first aluminum electrode tried, consisted of a wire ground to a fine point as it was assumed that this would allow enough current to pass to operate a direct-current voltmeter. For the other electrode a carbon rod of an arc lamp was used, and the electrolyte used was an aqueous solution of ammonium phosphate. With this arrangement, when the cell and voltmeter were connected in series across a line whose potential difference was 110 volts, the reading of the direct current voltmeter depended on the depth that the aluminum electrode was immerged in the electrolyte. The highest reading, about 19 volts, was obtained when the electrode was adjusted so that it just touched the electrolyte. By putting a shunt of comparatively low resistance across the terminals of the voltmeter, a reading of about 35 volts was obtained. As the per-cent of current rectified is proportional to the voltmeter reading, the conclusion was arrived at that the rectifying properties of the cell varied with the current density in the aluminum electrode.

Wave forms of the alternating-current voltage and of the rectified current were taken by means of a Hospitalier ondograph. Different electrolytes were tried such as citric acid, sulphuric acid, potassium acid tartrate, and sodium

acid tartrate solutions. Of these, sodium acid tartrate gave the best results in the percent of current rectified and it was used thruout the experiment. After numerous experiments in determining the proper amount of resistance to put in series with the cell to ensure the proper current density, very good results were obtained in rectifying the current as shown in Fig. 1. Difficulty, however, was experienced in keeping the value of the rectified voltage constant and it was found impossible to take the rectified wave to the same scale as the alternating-current voltage. More constant results were obtained by using a small plate of sheet aluminum, and because the electrolyte acted on the carbon, and held small particles of it in suspension, the carbon electrode was exchanged for lead.

After numerous experiments the results shown in Fig. 2 were obtained. The aluminum electrode used, when these curves were taken, was about 4 sq. inches in area, and the other electrode consisted of two lead plates each larger than the aluminum electrode and fastened on each side of it, leaving a space of about one-eight inch between each sheet of lead and the sheet of aluminum. The lead plates were electrically connected and in this way a short path thru the electrolyte was ensured for the current . To connect the aluminum electrode to the electric circuit the aluminum plate was cut so that a narrow strip protruded out of the electrolyte and this strip was insulated by a rubber tube

to prevent leakage at the surface of the electrolyte. A shunt of 1.54 ohms was used across the voltmeter so that a sufficient amount of current would flow.

With the above electrodes the same electrolytes were tried as with the aluminum point and the solution of sodium acid tartrate was again found to give the best results. Under certain conditions the direct-current reading could be brought up to 42 volts when the alternating-current voltage was 111.0 volts but could not be kept at that high value. After numerous experiments under different conditions, the conclusion was arrived at that, with the conditions in the alternating-circuit remaining constant, the direct-current voltmeter reading depended on

1. The electrolyte used.
2. The resistance in the shunt across the voltmeter.
3. The temperature of the electrolyte.
4. The time that the aluminum electrode was used.

It was found that with all other conditions constant the voltmeter reading fell after the aluminum electrode had been in use about an hour and that by renewing the aluminum, after about five minutes, the time necessary to form the insulating film, the direct-current voltmeter reading would rise to its first value. This is contrary to the experience of others and the reason for it is unknown. Most authorities state that the film should be allowed to form one or two hours to a day, but when this was tried the results were

not so good as when the film was formed in five minutes.

To determine whether the direct-current voltmeter would give readings in proportion to the mean value of the ordinates of the alternating-current voltage waves when the value of the voltage and the form of the voltage waves were changed, the wave forms shown in Fig. 3 and Fig. 4 were taken. It was found less difficult to keep the reading of the direct-current voltmeter constant at values lower than those which could be obtained under certain conditions. Before the curves in Fig. 3 were taken, the reading was allowed to drop to 36 volts when the alternating-current voltage was 111.0 volts and was kept constant at that point by keeping the temperature constant by packing the cell in ice. The curves in Fig. 3 were takon for these voltage values and the curves in Fig. 4 were taken by changing the connections of the cell to another line on which the voltage was 54.5 volts. In this case the reading of the direct-current voltmeter was 17.6 volts. To insure that no change had taken place in the cell it was immediately changed back to the first line after the curves had been taken and when the direct-current voltmeter reading was compared with its first value the readings were found to check.

Results.

Alternating-current voltage=111.0 volts.

Direct-current voltmeter reading=36.0 volts.

Alternating-Current Voltage Wave			Rectified-Current Voltage Wave		
Deg.	y	y^2	y_1	Deg.	y_1
0	0	0	-	182.5	4.9
2.5	2.0	4.0	-0	187.5	6.0
7.5	5.0	25.0	-1.5	192.5	7.5
12.5	8.5	72.25	-2.4	197.5	9.2
17.5	12.0	144.00	-3.2	202.5	11.8
22.5	16.3	265.69	-4.0	207.5	15.0
27.5	20.0	400.00	-4.2	212.5	19.0
32.5	22.5	506.25	-4.8	217.5	21.9
37.5	25.0	625.00	-4.8	222.5	24.0
42.5	28.5	812.25	-4.8	227.5	27.2
47.5	32.0	1024.00	-5.1	232.5	30.0
52.5	34.4	1183.36	-5.2	237.5	32.1
57.5	36.0	1296.00	-6.0	242.5	33.5
62.5	38.0	1444.00	-6.1	247.5	34.9
67.5	40.5	1640.25	-6.1	252.5	36.2
72.5	41.5	1722.25	-6.2	257.5	37.0
77.5	42.4	1806.25	-7.0	262.5	37.0
82.5	43.0	1849.00	-7.1	267.5	37.2
87.5	43.5	1892.25	-8.0	272.5	37.5

A. C. wave

Σy=1603.6

Meany=27.12

Σy^2=39532.09

M.R.S.=30.55

Rectified wave

Σy_1=724.1

Meany=10.05

(Continued)

Alternating-Current Voltage Wave			Rectified-Current Voltage Wave		
Deg.	y	y^2	y_1	Deg.	y_1
92.5	44.0	1936.00	-8.1	277.5	37.3
97.5	44.0	1936.00	-8.0	282.5	36.5
102.5	43.2	1866.24	-8.0	287.5	35.4
107.5	42.5	1806.25	-7.2	292.5	35.4
112.5	41.2	1697.44	-7.0	297.5	33.0
117.5	39.9	1592.01	-6.3	302.5	31.5
122.5	37.8	1428.84	-5.3	307.5	29.9
127.5	36.0	1296.00	-4.1	312.5	27.1
132.5	33.0	1089.00	-3.0	317.5	24.0
137.5	29.2	852.64	-1.6	322.5	22.0
142.5	26.1	681.21	-0.6	327.5	20.0
147.5	23.8	566.44	-0.5	332.5	17.3
152.5	20.0	400.00	-1.0	337.5	14.8
157.5	17.8	316.84	-1.4	342.5	11.8
162.5	14.1	198.81	-2.6	347.5	8.2
167.5	10.0	100.00	-2.8	352.5	5.0
172.5	6.9	47.61	-3.9	357.5	2.5
177.5	3.0	9.00	-4.5	360.0	1.5
180.0	0.0	0 0			

Alternating-Current Voltage Wave.

Effective value, 54.5 volts.

Deg.	y	y^2	Deg.	y	y^2
0	0	0	92.5	21.5	462.25
2.5	1.0	1.0	97.5	21.2	449.44
7.5	2.9	8.41	102.5	21.0	441.00
12.5	4.5	22.50	107.5	20.3	412.09
17.5	6.2	38.44	112.5	19.9	396.01
22.5	7.2	51.81	117.5	19.0	361.00
27.5	9.0	81.00	122.5	18.0	324.00
32.5	11.0	132.00	127.5	16.9	285.61
37.5	12.2	148.84	132.5	15.6	243.36
42.5	13.9	193.21	137.5	14.0	196.00
47.5	15.0	225.00	142.5	13.0	169.00
52.5	16.7	278.89	147.5	11.2	125.44
57.5	17.8	316.84	152.5	10.0	100.00
62.5	18.3	334.89	157.5	8.2	67.24
67.5	19.5	380.25	162.5	6.5	42.25
72.5	20.0	400.00	167.5	4.2	16.16
77.5	21.0	441.00	172.5	2.6	6.76
82.5	21.1	445.21	177.5	1.3	1.69
87.5	21.7	470.89	180.0	0.0	0.00

$\Sigma y = 483.4$ $\Sigma y^2 = 8069.68$

Mean $y = 13.065$ M. R. S. $= 14.77$

Rectified-Current Voltage Wave.

Mean value, 17.6 volts.

Effective value of impressed voltage.

Degrees	y_1	Degrees	y_1
2.5	0	97.5	-1.8
7.5	-0.5	102.5	-1.8
12.5	-1.1	107.5	-1.7
17.5	-1.6	112.5	-1.1
22.5	-1.9	117.5	-1.0
27.5	-2.0	122.5	-0.5
32.5	-2.0	127.5	0
37.5	-2.0	132.5	0
42.5	-2.1	137.5	0.5
47.5	-2.1	142.5	1.0
52.5	-2.1	147.5	1.1
57.5	-2.0	152.5	1.5
62.5	-2.0	157.5	1.7
67.5	-2.0	162.5	2.0
72.5	-2.0	167.5	2.2
77.5	-2.0	172.5	2.5
82.5	-1.8	177.5	2.7
87.5	-1.8	182.5	3.0
92.5	-1.8	187.5	3.1
		192.5	3.9

Rectified-Current Voltage Wave. (continued)

Mean value, 17.6 volts.

Effective value of impressed voltage.

Degrees	y_1	Degrees	y_1
197.5	4.6	282.5	16.9
202.5	6.0	287.5	16.5
207.5	6.9	292.5	15.2
212.5	8.4	292.5	14.8
217.5	9.9	302.5	14.0
222.5	10.7	307.5	13.0
227.5	12.0	312.5	11.9
232.5	13.0	317.5	10.3
237.5	13.9	322.5	9.3
242.5	14.6	327.5	8.0
247.5	15.1	332.5	7.0
252.5	16.0	337.5	5.0
257.5	16.5	342.5	3.9
262.5	17.0	347.5	2.4
267.5	17.1	352.5	1.2
272.5	17.0	357.5	1.0
277.5	16.9	362.5	0

$\Sigma y_1 = 347.5$ Mean $y_1 = 4.82$

$E = 111, \quad E_a = 36$

M. R. S of Ordinates = 30.55

Average Ordinate = 27.12

Average Ordinate Rectified wave = 10.05

$$K = \frac{111}{30.55} = 3.633$$

$$y = 27.12 \times 3.633 = 98.53 \text{ volts.}$$

$$K_1 = \frac{98.53}{36} = 2.737$$

$$\text{Form factor of wave} = \frac{111}{98.53} = 1.126$$

$$\frac{Bo}{B} = \frac{0.9 \times 111}{98.53} = 1.0149$$

$$K \times \text{mean ordinate of rectified wave} = 3.633 \times 10.05$$

$$= 36.51$$

$E = 54.5 \quad E_a = 17.6$

M. R. S. of Ordinates = 14.77

Average ordinate = 13.065

Average ordinate of rectified wave = 4.82

$$K = \frac{54.5}{14.77} = 3.689$$

$$y = 13.065 \times 3.688 = 48.196 \text{ volts.}$$

$$K_1 = \frac{48.196}{17.5} = 2.739$$

$$\text{Form factor of wave} = \frac{54.5}{48.184} = 1.131$$

$$Bo/B = \frac{.9 \times 54.5}{48.196} = 1.0179$$

K x mean ordinate of rectified wave = 17.77

Conclusion

From the results obtained it is seen that the mean value of the alternating current can be calculated from the direct-current voltmeter reading, for if the reading obtained in the second case were multiplied by the coefficient, K_1 obtained in the first case, the mean value of the voltage would become 48.07 instead of 48.196 as calculated. This is as accurate a result as can be expected as it is not possible to read the voltmeter closer than 0.1 volt. The mean ordinate of the rectified wave multiplied by K should equal the direct-current voltmeter reading but in both cases it is slightly larger. This however does not effect the results, it only shows that the waves of alternating-current voltage and rectified current were not taken to the same scale.

Unless some means could be devised so that the direct-current voltmeter would give a constant reading when connected in series with the rectifying cell, across a line with conditions in the line constant, and give the same reading every time, this method of determining the mean ordinate of voltage waves is not practical, because, for every change in the reading of the voltmeter, caused by a change of condition in the cell, the coefficient, K , changes, and when

applied to another circuit it is impossible to tell whether the conditions in the cell have remained constant or not.

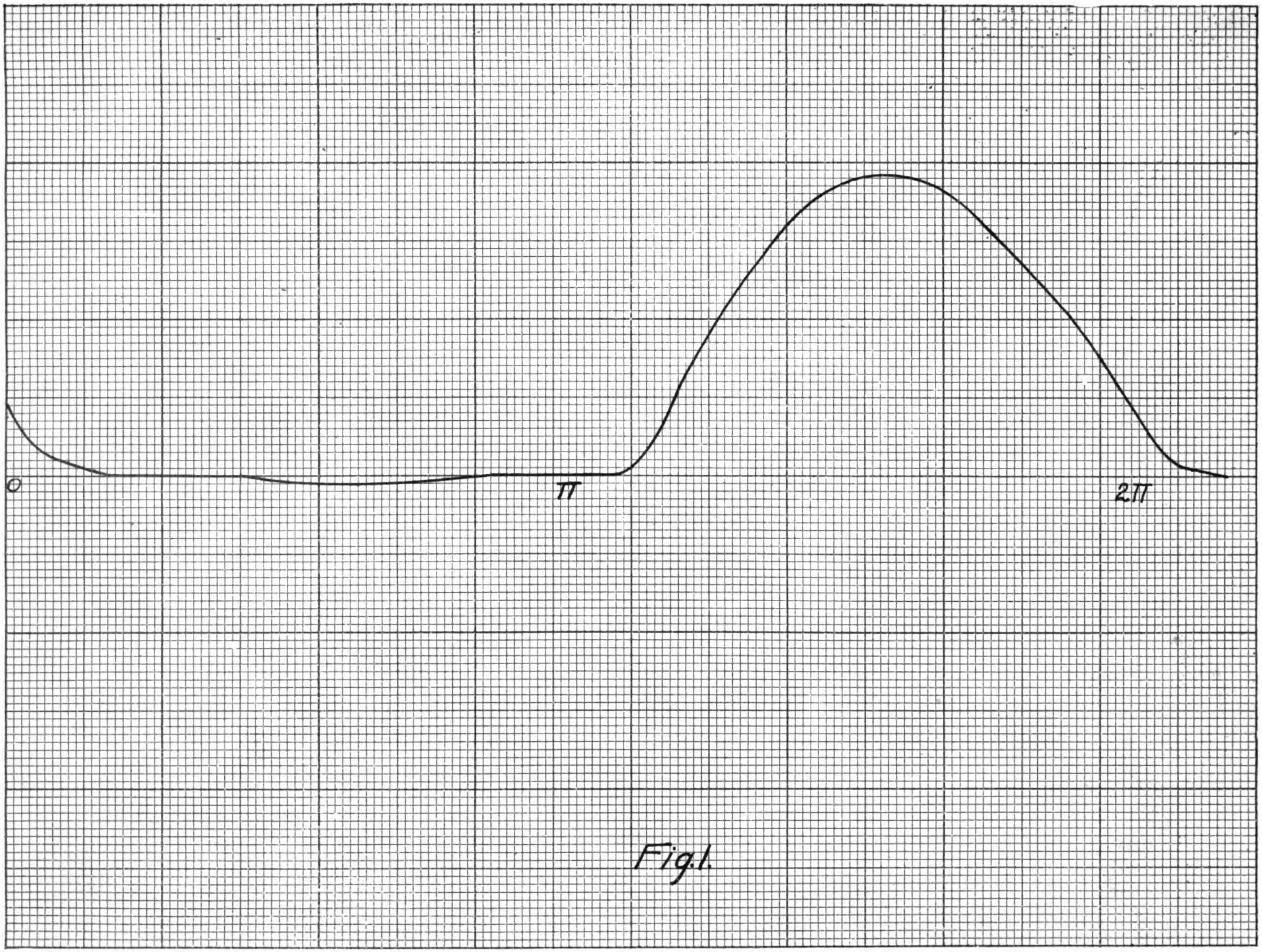

Fig.I.

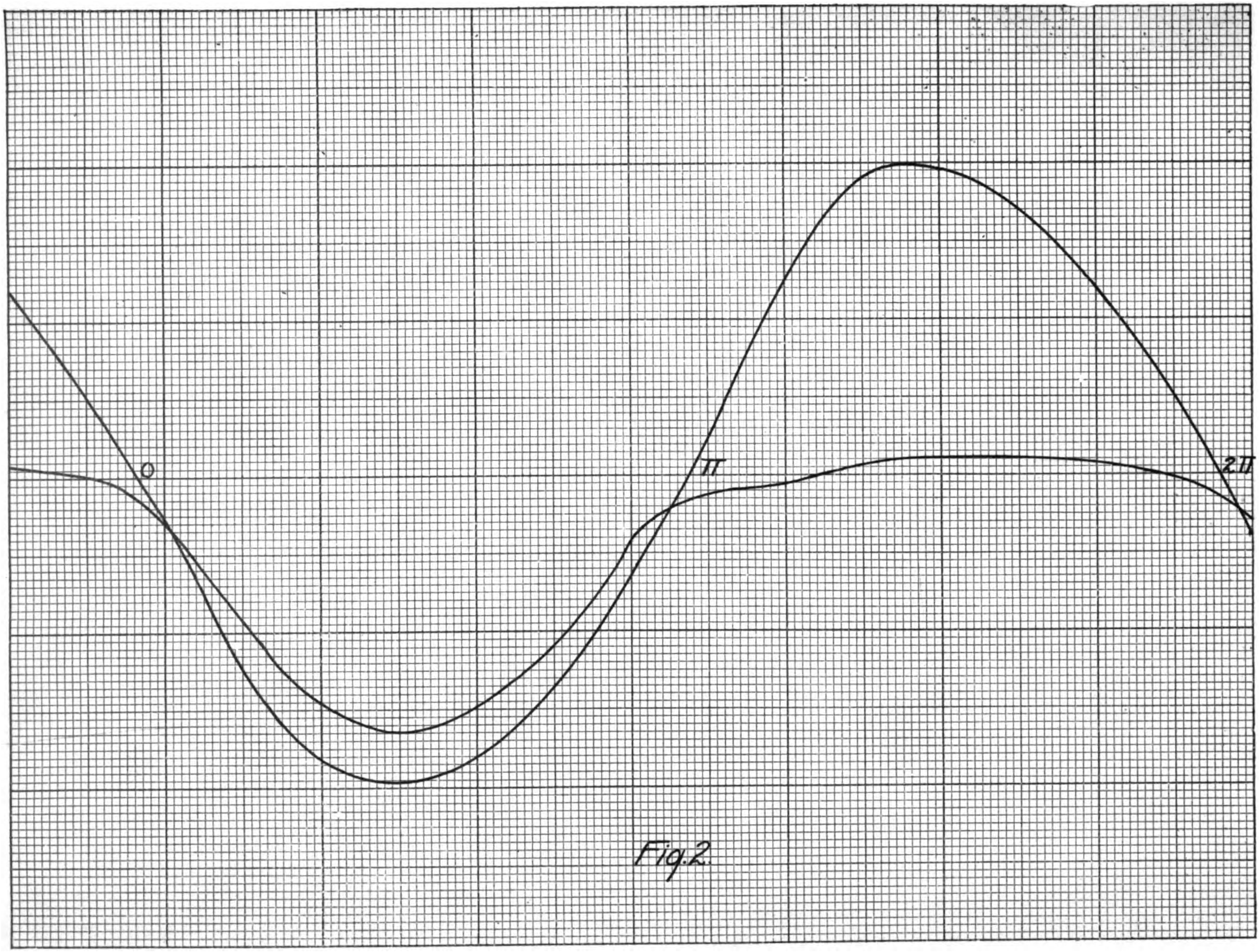

Fig. 2.

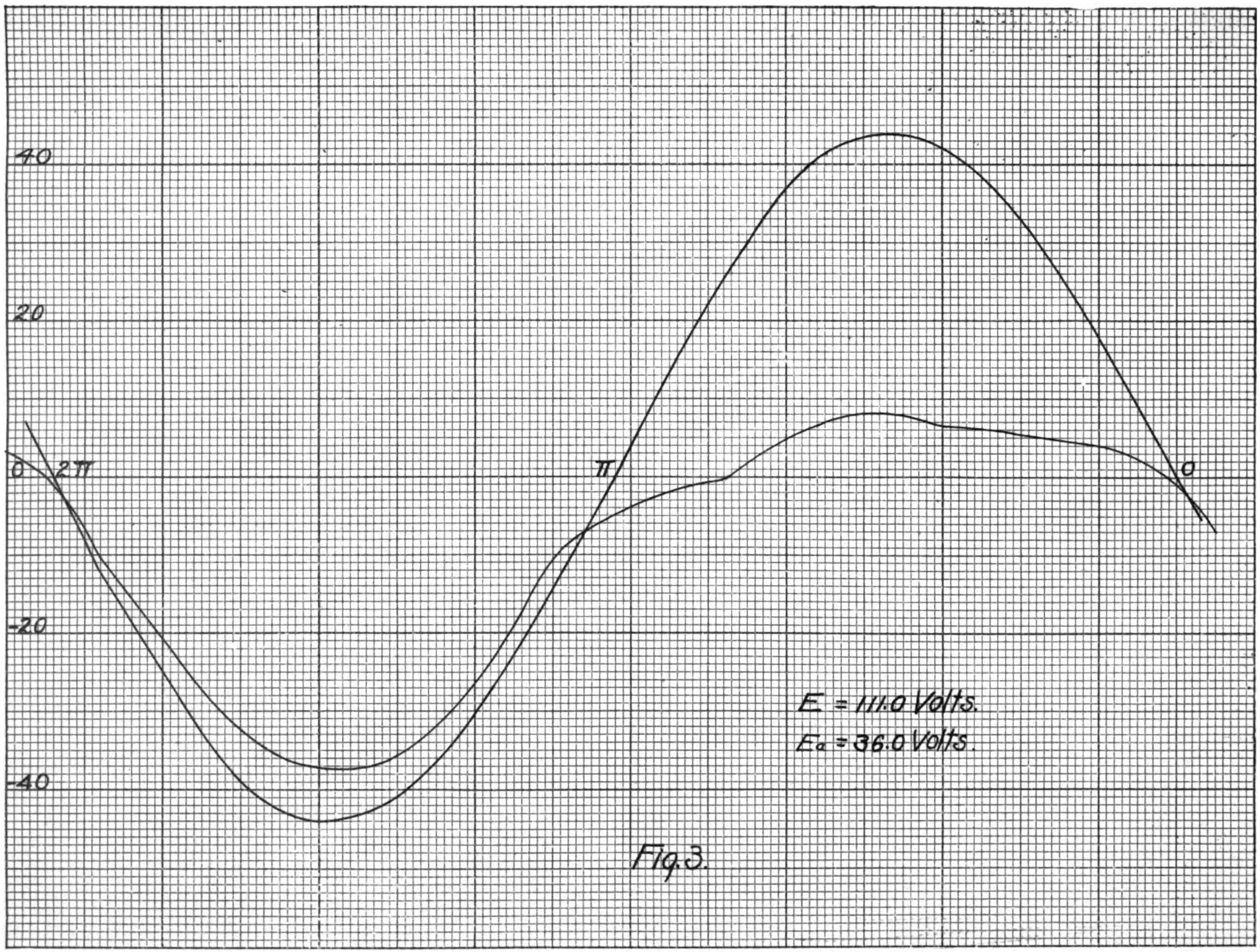

Fig.3.

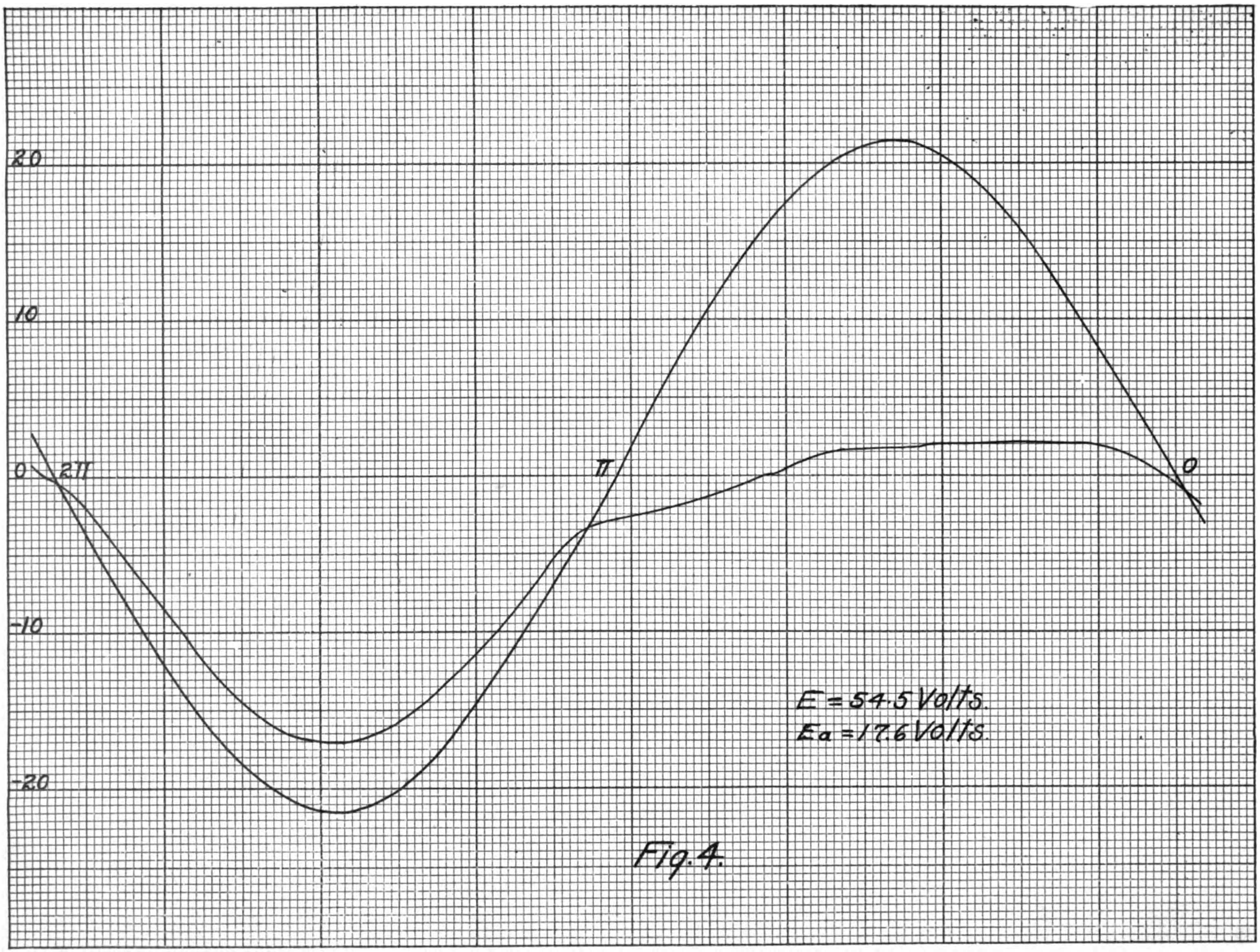

Fig. 4.

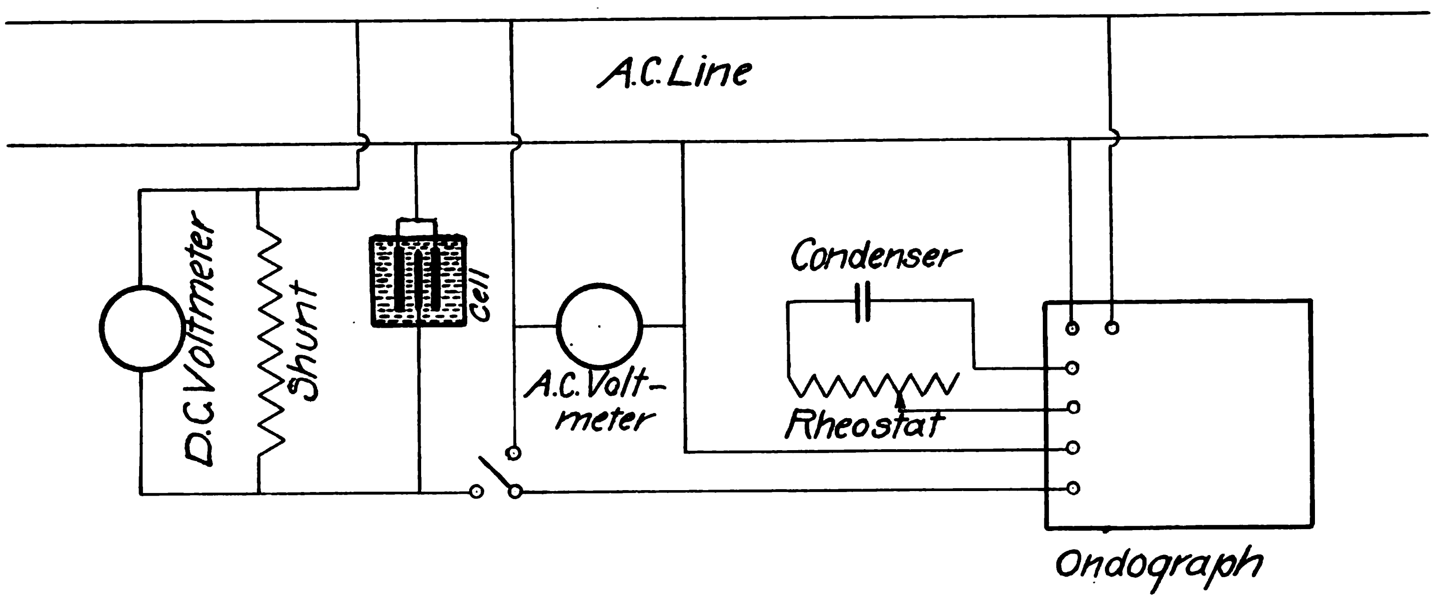

Fig. 5. Diagram of Connections.

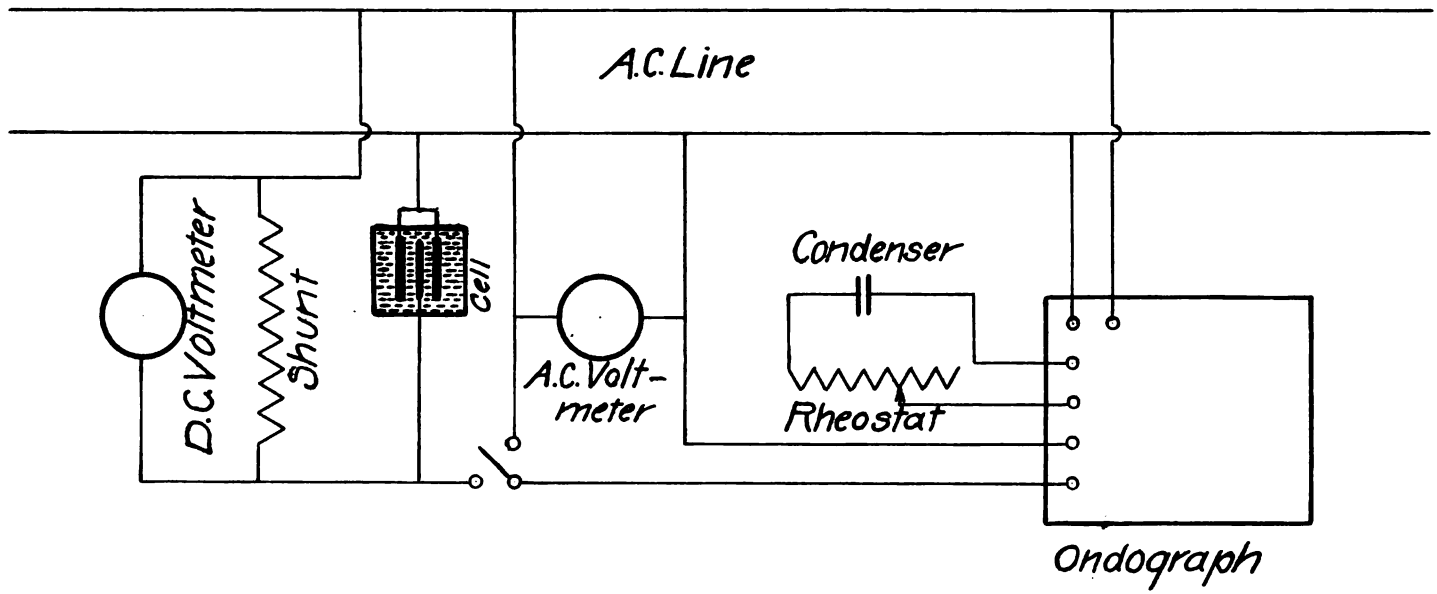

Fig. 5. Diagram of Connections.

Approved

M C Beebe

Prof. Elec. Eng

June 16, '11